# Crystal Structure of $\mathrm{Na}_{3} \mathrm{PO}_{\mathbf{4}} \cdot \frac{1}{2} \mathrm{H}_{2} \mathrm{O}$ 

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#### Abstract

The crystal structure of trisodium monophosphate hemihydrate was determined. The space group is $C 2 / c$ and a unit cell contains eight formula units. The unit cell dimensions of $\mathrm{Na}_{3} \mathrm{PO}_{4} \cdot \frac{1}{2} \mathrm{H}_{2} \mathrm{O}$ are $a=$ $9.631(3), b=5.416(2), c=16.938(8) \AA, \beta=102.60(5)^{\circ}$. The final $R$ value is 0.027 for a set of 1430 independent reflections. This atomic arrangement is mainly a three-dimensional network of distorted $\mathrm{NaO}_{6}$ octahedra. The hydrogen bonding scheme is given.


## Introduction

Trisodium monophosphate hemihydrate has been characterized during various investigations of the $\mathrm{H}_{2} \mathrm{O}-\mathrm{P}_{2} \mathrm{O}_{5}-\mathrm{Na}_{2} \mathrm{O}$ system (1-4), but up to now the crystal structure of this salt has not been investigated.

## Chemical Preparation

$\mathrm{Na}_{3} \mathrm{PO}_{4} \cdot \frac{1}{2} \mathrm{H}_{2} \mathrm{O}$ crystals were obtained during experiments run for the preparation of $\mathrm{Na}_{3} \mathrm{PO}_{4}$ (H.T.) crystals. Schematically the reaction used here is

$$
\mathrm{AlPO}_{4}+4 \mathrm{NaOH} \rightarrow \mathrm{Na}_{3} \mathrm{PO}_{4}+\mathrm{NaAlO}_{2}
$$

The aluminum monophosphate is dissolved in a concentrated sodium hydroxide solution which is then slowly evaporated at low temperature $\left(60-80^{\circ} \mathrm{C}\right)$. Crystals of $\mathrm{Na}_{3} \mathrm{PO}_{4} \cdot \frac{1}{2} \mathrm{H}_{2} \mathrm{O}$ appear when the temperature is approximately $60^{\circ} \mathrm{C}$. They are stout monoclinic prisms, apparently very stable at room temperature since their preparation more than 2 months ago.

## Crystal Data and Structure Determination

$\mathrm{Na}_{3} \mathrm{PO}_{4} \cdot \frac{1}{2} \mathrm{H}_{2} \mathrm{O}$ is monoclinic with the unit cell dimensions $a=9.631(3), b=$ $5.416(2), c=16.938(8) \AA, \beta=102.60(5)^{\circ}$. There are eight formula units per cell and the calculated density is 2.657 . The observed extinction conditions

$$
\begin{aligned}
& h k l \text { with } h+k=2 n, \\
& h 0 l \text { with } h=2 n \text { and } 1=2 n
\end{aligned}
$$

correspond to $C c$ or $C 2 / c$ as possible space groups. The structure determination will show that the centrosymmetrical $C 2 / c$ is the correct one.

A prismatic crystal $(0.32 \times 0.26 \times 0.26$ $\mathrm{mm}^{3}$ ) was chosen for the measurements on a Philips PW 1100 four-circle automatic diffractometer operating with silver $K \alpha$ radiation ( $0.5608 \AA$ ) monochromatized by a graphite plate. The intensities of 1755 reflections with $\theta<30^{\circ}$ were measured, using the following conditions: $\omega$-scan, scan speed $0.03^{\circ} \mathrm{sec}^{-1}$, scan width $1.20^{\circ}$. Back-

TABLE I
Atomic Coordinates

| Atom | $x(\sigma)$ | $y(\sigma)$ | $z(\sigma)$ | $B_{\text {eq. }}(\sigma)$ |
| :--- | :--- | :--- | :--- | :--- |
| $\mathbf{P}$ | $0.15691(4)$ | $0.38662(7)$ | $0.11307(2)$ | $0.700(5)$ |
| $\mathrm{Na}(1)$ | $0.18100(8)$ | $0.5305(1)$ | $0.95193(4)$ | $1.38(1)$ |
| $\mathrm{Na}(2)$ | $0.00084(9)$ | $0.0973(2)$ | $0.59730(5)$ | $1.83(1)$ |
| $\mathrm{Na}(3)$ | $0.64975(7)$ | $0.1680(1)$ | $0.78490(4)$ | $1.22(1)$ |
| $\mathrm{O}(\mathrm{W})$ | $0.0000(0)$ | $0.9551(3)$ | $0.25000(0)$ | $1.63(3)$ |
| $\mathrm{O}(1)$ | $0.1013(1)$ | $0.2354(2)$ | $0.03580(7)$ | $1.28(2)$ |
| $\mathrm{O}(2)$ | $0.0324(1)$ | $0.5265(2)$ | $0.64902(7)$ | $1.29(2)$ |
| $\mathrm{O}(3)$ | $0.7564(1)$ | $0.2800(2)$ | $0.67431(7)$ | $1.31(2)$ |
| $\mathrm{O}(4)$ | $0.7412(1)$ | $0.1124(2)$ | $0.09349(7)$ | $1.32(2)$ |
| H | $0.429(3)$ | $0.357(6)$ | $0.229(2)$ | $3.1(7)$ |

Note. The estimated standard deviations are given in parentheses. Thermal factors are $\boldsymbol{B}_{\text {eq. }}$ for nonhydrogen atoms and $B_{\text {iso }}$. for hydrogen atoms.
syntheses. After some refinement cycles with anisotropic thermal parameters the $R$ value is 0.028 for a set of 1430 reflections such that

$$
\begin{aligned}
& F_{0}>2 \sigma_{F}, \\
& F_{0}-F_{c}<20 \quad \text { in a scale } 0-1026 .
\end{aligned}
$$

At this stage a difference Fourier map revealed the hydrogen atoms. Final refinement cycles, including the hydrogen atoms (with isotropic thermal factors), gave a final $R$ value of 0.027 with the same set of reflections.

Table I reports the final atomic coordinates with the calculated $B_{\text {eq. }}$ for nonhydrogen atoms and $B_{\text {iso. }}$ for hydrogen atoms. All atoms are in the general position of the $C 2 / c$ space group with the exception of the water

TABLE II
Anisotropic Thermal Parameters $\beta_{\mathrm{ij}}$ for Nonhydrogen Atoms

| Atom | $\beta_{11}$ | $\beta_{22}$ | $\beta_{33}$ | $\beta_{12}$ | $\beta_{13}$ | $\beta_{23}$ |
| :--- | :--- | :--- | :--- | :--- | :--- | ---: |
| $\mathbf{P}$ | $0.00201(2)$ | $0.00548(7)$ | $0.00068(1)$ | $-0.0004(1)$ | $0.00050(2)$ | $0.00006(6)$ |
| $\mathrm{Na}(1)$ | $0.00498(6)$ | $0.0105(2)$ | $0.00098(2)$ | $-0.0046(2)$ | $0.00042(6)$ | $0.0012(1)$ |
| $\mathrm{Na}(2)$ | $0.00688(8)$ | $0.0122(2)$ | $0.00142(2)$ | $0.0057(2)$ | $0.00098(7)$ | $-0.0003(1)$ |
| $\mathrm{Na}(3)$ | $0.00377(6)$ | $0.0110(2)$ | $0.00092(2)$ | $0.0008(2)$ | $0.00072(5)$ | $0.0003(1)$ |
| $\mathrm{O}(\mathrm{W})$ | $0.00397(15)$ | $0.0081(4)$ | $0.00217(6)$ | $0.0000(0)$ | $0.00031(16)$ | $0.0000(0)$ |
| $\mathrm{O}(1)$ | $0.00512(11)$ | $0.0090(3)$ | $0.00085(3)$ | $-0.0049(3)$ | $0.00074(9)$ | $-0.0011(2)$ |
| $\mathrm{O}(2)$ | $0.00283(9)$ | $0.0149(3)$ | $0.00110(3)$ | $-0.0037(3)$ | $0.00129(8)$ | $-0.0008(2)$ |
| $\mathrm{O}(3)$ | $0.00343(9)$ | $0.0111(3)$ | $0.00125(3)$ | $-0.0043(3)$ | $0.00052(9)$ | $-0.0021(2)$ |
| $\mathrm{O}(4)$ | $0.00499(10)$ | $0.0092(3)$ | $0.00098(3)$ | $-0.0070(3)$ | $0.00075(9)$ | $0.0003(2)$ |

Note. Estimated standard deviations are given in parentheses. The formula used here is

$$
T=\beta_{11} h^{2}+\beta_{22} k^{2}+\beta_{33} l^{2}+\beta_{12} h k+\beta_{13} h l+\beta_{23} k l
$$

ground was measured during 5 sec at each extremity of the scan domain. Two reference reflections $\overline{1} 36$ and $1 \overline{3} \overline{6}$ ) were measured every 2 hr without any significant variation. A final set of 1715 independent observations was obtained from this measurement. No absorption correction was made.
The crystal structure was solved by using classical methods: study of the Patterson function followed by successive Fourier

TABLE III
Main Interatomic Distances and Bond Angles in the $\mathrm{PO}_{4}$ Tetrahedron

| P | $\mathrm{O}(1 a)$ | $\mathrm{O}(2 e)$ | $\mathrm{O}(3 g)$ | $\mathrm{O}(4 e)$ |
| :---: | :--- | :--- | :--- | :--- |
| $\mathrm{O}(1 a)$ | $1.540(2)$ | $2.521(2)$ | $2.498(2)$ | $2.524(2)$ |
| $\mathrm{O}(2 e)$ | $110.24(10)$ | $1.533(2)$ | $2.513(2)$ | $2.515(2)$ |
| $\mathrm{O}(3 g)$ | $108.23(10)$ | $109.61(9)$ | $1.542(2)$ | $2.517(2)$ |
| $\mathrm{O}(4 e)$ | $109.82(9)$ | $109.64(10)$ | $109.28(10)$ | $\underline{1.544(2)}$ |

TABLE IV
Main Interatomic Distances and Bond Angles in the $\mathrm{NaO}_{6}$ Polyhedra

| $\mathrm{Na}(1) \mathrm{O}_{6}$ polyhedron |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathrm{Na}(1)$ | O(1a) | O(1d) | O(2f) | O(3f) | $\mathrm{O}(4 b)$ | $\mathrm{O}(4 \mathrm{c})$ |
| O(1a) | 2.377(2) | 3.350(4) | 3.498(2) | 4.099(2) | 4.588(2) | 2.524(2) |
| O(1d) | 86.42(7) | 2.514(2) | 4.412(2) | 2.498(2) | 3.686(3) | 3.482(2) |
| O(2f) | 94.86(7) | 129.05(7) | 2.372(2) | 3.088(2) | 3.381(2) | 4.403(2) |
| O(3f) | 107.07(7) | 56.92(5) | 74.41 (6) | 2.714(2) | 3.556(2) | 4.896(2) |
| O(4b) | 161.91(8) | 100.67(7) | 93.47(7) | 90.62(7) | 2.269(2) | 3.545(3) |
| $\mathrm{O}(4 c)$ | 63.95(6) | 90.46(7) | 135.31(7) | 147.25(7) | 99.10(6) | 2.388(2) |
| $\mathrm{Na}(2) \mathrm{O}_{6}$ polyhedron |  |  |  |  |  |  |
| Na(2) | O(W) | $\mathrm{O}(1 e)$ | $\mathrm{O}(1)$ | $\mathrm{O}(2 a)$ | $\mathrm{O}(4 e)$ | $\mathrm{O}(4 \mathrm{~g})$ |
| O(W) | 2.611(1) | 4.247(2) | 4.849(2) | 3.174(3) | 3.334(2) | 4.313(2) |
| O(1e) | 116.18(8) | 2.391(2) | 3.279(3) | 4.661(2) | 3.862(2) | 3.686(3) |
| $\mathrm{O}(1)$ | 152.53(7) | 86.84(7) | 2.380(2) | $3.498(2)$ | 3.482(2) | 3.636(3) |
| O(2a) | 77.11(7) | 146.21(8) | 92.02(7) | 2.480 (2) | 4.425 (2) | 2.515(2) |
| O(4e) | 77.13(5) | 97.57(7) | 85.50(6) | 116.02(6) | 2.734(2) | 5.525(0) |
| O(4g) | 105.36(6) | 89.86(7) | 88.53(7) | 56.35(5) | 170.18(8) | 2.811(2) |
| $\mathrm{Na}(3) \mathrm{O}_{6}$ polyhedron |  |  |  |  |  |  |
| $\mathrm{Na}(3)$ | O(W) | $\mathrm{O}(2 \mathrm{c})$ | O(2h) | $\mathrm{O}(3 a)$ | O(3h) | $\mathrm{O}(4 d)$ |
| O(W) | $2.495(2)$ | 3.340(3) | 3.340(3) | 3.346(2) | 4.796(3) | 3.334(2) |
| O(2c) | 84.76(6) | 2.460 (2) | 3.618(3) | 2.513(2) | 3.501(2) | 4.852(2) |
| O(2h) | 85.92(6) | 96.07(7) | 2.405(2) | 4.719(2) | 3.088(2) | 3.381(2) |
| O(3a) | 86.05(5) | 62.17(6) | 157.42(7) | 2.407(2) | 3.755(2) | 3.981(2) |
| O(3a) | 166.95(7) | 93.82(7) | 81.32(7) | 104.77(6) | 2.333(2) | 3.556(2) |
| O(4d) | 85.50(6) | 168.64(7) | 89.05 (6) | 111.30(7) | 96.98(7) | 2.416(2) |

TABLE V
Hydrogen Bond Scheme (Angles and Distances)

| $\mathrm{O}(\mathrm{W})-\mathrm{H}$ | $\mathrm{H} . \ldots(3)$ | $\mathrm{O}(\mathrm{W})-\mathrm{O}(3)$ | $\mathrm{H}-\mathrm{O}(\mathrm{W})-\mathrm{H}$ | $\mathrm{O}(\mathrm{W})-\mathrm{H} . \ldots . \mathrm{O}(3)$ |
| :---: | :---: | :---: | :---: | :---: |
| $(2 \times) 0.88(4) \AA$ | $(2 \times) 1.87(4) \AA$ | $2.732(2) \AA$ | $106(5)^{\circ}$ | $166(4)^{\circ}$ |

TABLE VI
Symmetry Code Used in Tables III and IV
molecule located on a twofold axis. Table II gives the anisotropic thermal factors for the nonhydrogen atoms.
A unitary weighting scheme was used for all least-squares calculations.

## Description of the Structure

The $\mathrm{PO}_{4}$ tetrahedron, whose main interatomic distances and bond angles are re-


Fig. 1. Projection of the atomic arrangement along the $\mathbf{b}$ axis.
ported in Table III, is not very distorted:

$$
\begin{array}{r}
1.533<\mathrm{P}-\mathrm{O}<1.544 \AA \\
108.23<\mathrm{O}-\mathrm{P}-\mathrm{O}<110.24^{\circ}
\end{array}
$$

with averages $\overline{\mathrm{P}-\mathrm{O}}=1.540 \AA$ and $\mathrm{O} \widehat{-\mathrm{P}-\mathrm{O}}$ $=109.47^{\circ}$.
$\mathrm{NaO}_{6}$ polyhedra. The three independent sodium atoms have a strongly distorted octahedral coordination. The main geometrical features of these polyhedra are reported in Table IV.

The water molecule and the hydrogen bond scheme. The water molecule is located on a twofold axis. Table IV reports the main characteristics for this molecule and the hydrogen bond scheme.

As can be seen from Fig. 1, this atomic arrangement may be described as a three-
dimensional network of very distorted $\mathrm{NaO}_{6}$ octahedra.

In addition, it may be noticed that $\mathrm{NaO}_{6}$ octahedra are not equivalent. $\mathrm{Na}(1) \mathrm{O}_{6}$ polyhedron is built up only with oxygen atoms, while $\mathrm{Na}(2) \mathrm{O}_{6}$ and $\mathrm{Na}(3) \mathrm{O}_{6}$ polyhedra have a water molecule in their coordination. The average values for the $\mathrm{Na}-\mathrm{O}$ distances in these three octahedra are, respectively, $2.439\left(\mathrm{Na}_{1}\right), 2.568\left(\mathrm{Na}_{2}\right)$, and $2.419\left(\mathrm{Na}_{3}\right)$.

## References

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